RATIOS, RATES AND PROPORTIONS

Ratios

Quantities such as 8 feet, 16 cents or 10 hours are numerical quantities written with units. A **ratio** is a comparison of two quantities with the ***same*** units. For example, to compare the heights of two trees, one 6 feet (ft) tall and the other 8 feet (ft) tall, we can write this ratio three ways:

1) As a **fraction**: $\frac{6 \cancel{\text{ft}}}{8 \cancel{\text{ft}}} = \frac{6}{8} = \frac{\cancel{2} \bullet 3}{\cancel{2} \bullet 4} = \frac{3}{4}$

2) With a **colon**: 6 ft **:** 8 ft = 6 **:** 8 = 3 **:** 4

3) With the word **to**: 6 ft to 8 ft. = 6 to 8 = 3 to 4.

In a ratio, the ***order*** of the numbers is important! To write a ratio as a fraction,

- Place the ***first*** number in the ***numerator***; place the ***second*** number in the ***denominator***.
- Write the ratio in lowest terms – cancel common factors.
- A ratio compares two quantities with the ***same*** units. Convert the units if they are not the same.

Example 1: Write each ratio as a fraction in lowest terms.

a) $1\frac{1}{2}$ miles (mi) to $2\frac{1}{4}$ miles (mi)

To write the ratio as a fraction in lowest terms, divide the numerator by the denominator and cancel common factors:

$$1\frac{1}{2}\text{ mi to }2\frac{1}{4}\text{ mi} = \frac{1\frac{1}{2}}{2\frac{1}{4}} = 1\frac{1}{2} \div 2\frac{1}{4} = \frac{3}{2} \div \frac{9}{4} = \frac{{}^{1}\cancel{3}}{{}_{1}\cancel{2}} \bullet \frac{\cancel{4}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$$

b) 75 cents to \$1.25

The units are *not* the same. Since \$1 = 100 cents, convert \$1.25 to cents by moving the decimal point two places to the right: \$1.25 = 1.25 = 125 cents. This gives us:

$$75\text{ cents to }\$1.25 = 75\text{ cents to }125\text{ cents} = \frac{75}{125} = \frac{3 \bullet \cancel{25}}{5 \bullet \cancel{25}} = \frac{3}{5}$$

RATES

A **rate** is a comparison of two quantities with ***different*** units, such as the rate "10 grams per 180 milliliters." In a rate, the word "per" means "for each" and implies division. A rate is written as a fraction in lowest terms and the units are written as part of the rate. To write the rate 10 grams (g) per 180 milliliters (mL) as a fraction, cancel the corresponding 0's and keep the units:

$$10\text{ g per }180\text{ mL} = \frac{1\cancel{0}\text{ g}}{18\cancel{0}\text{ mL}} = \frac{1\text{ g}}{18\text{ mL}}$$

A **unit rate** or **unit price** is a rate where the denominator is "1." For example, if 1 pound of coffee costs \$6.75, the unit price is written as:

$$\frac{\$6.75}{1\,\text{lb}} = \$6.75/\text{lb or } \$6.75 \text{ per lb}$$

To find a unit rate or unit price, divide the numerator by the denominator.

Example 2: A car travels 330 miles on 15 gallons (gal). Find the unit rate.

To find the unit rate, we divide the number of miles by the number of gallons:

$$\text{330 miles on 15 gallons} = \frac{330\text{ mi}}{15\text{ gal}} = 15\overline{)330}^{\,22} = \frac{22\text{ mi}}{1\text{ gal}}$$

The unit rate is 22 mi/gal or 22 mpg.

PROPORTIONS

A **proportion** is a mathematical statement that two ratios or rates are equal. In a true proportion, the ***cross products*** are ***equal***:

$$\frac{3}{4} = \frac{6}{8} \qquad \begin{aligned} 4(6) &= 24 \\ 3(8) &= 24 \end{aligned}$$

Because the cross products are equal, we can use them to solve a proportion when one of the numbers is unknown. To solve a proportion,

1) Cross multiply the ratios
2) Write an equation by setting the cross products equal to each other
3) Solve the equation for the variable.

Example 3: Solve $\frac{4}{9} = \frac{12}{x}$

To solve the equation for x, find the cross products and solve the resulting equation:

$\frac{4}{9} = \frac{12}{x}$ ← Cross multiply the ratios.

$4 \cdot x = 9 \cdot 12$ ← Write an equation by setting the cross products equal to each other.

$4x = 108$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{108}{4}$ ← Solve the equation for x.

$x = 27$

SOLVING PROPORTION PROBLEMS

Proportions are used to solve a variety of problems, such as estimating wildlife populations, scaling distances on a map, or calculating mixtures and dosages.

Example 4: Use a proportion to solve the following problem:

> "The brewing directions on a bag of ground coffee recommend 2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water. If the average-size coffee cup holds 6 oz of coffee, how many tablespoons of coffee are needed for eight 6-oz cups?"

To write the proportion,

1) ***Identify the given ratio or rate and write it as a fraction.*** In the problem above, we are given the rate "2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water." We can write this rate as:

$$\text{2 tbs to 6 oz} = \frac{2\text{ tbs}}{6\text{ oz}} = \frac{1\text{ tbs}}{3\text{ oz}}$$

2) ***Assign a variable to represent the unknown quantity then write the second ratio or rate as a fraction.*** Let x = the number of tablespoons we need to make eight 6-oz cups. Since each cup holds 6 oz, to make eight 6-oz cups, we need 48 oz of water. We can write this second rate as:

$$\text{x tbs to 48 oz} = \frac{x\text{ tbs}}{48\text{ oz}}$$

3) ***Write the proportion by setting the first ratio or rate equal to the second.*** The order in which we write a proportion is important! *Always place like units across from each other.* Because the cross products are equal, inverting the ratios gives an *equivalent proportion*.

$$\frac{1\text{ tbs}}{3\text{ oz}} = \frac{x\text{ tbs}}{48\text{ oz}} \quad \begin{matrix}\leftarrow \text{tbs across from tbs} \\ \leftarrow \text{oz across from oz}\end{matrix} \quad \text{or} \quad \frac{3\text{ oz}}{1\text{ tbs}} = \frac{48\text{ oz}}{x\text{ tbs}} \quad \begin{matrix}\leftarrow \text{oz across from oz} \\ \leftarrow \text{tbs across from tbs}\end{matrix}$$

To find the number of tablespoons, cross multiply and solve the resulting equation:

$$\frac{1}{3} = \frac{x}{48}$$

$$3x = 48$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{48}{3}$$

$$x = 16\text{ tbs}$$

Hint: Alternatively, you may find it helpful to state the problem in the following way:

1 tbs is to 3 oz **as** x tbs is to 48 oz