

RATIOS, RATES AND PROPORTIONS

Ratios

Quantities such as 8 feet, 16 cents or 10 hours are numerical quantities written with units. A **ratio** is a comparison of two quantities with the **same** units. For example, to compare the heights of two trees, one 6 feet (ft) tall and the other 8 feet (ft) tall, we can write this ratio three ways:

- 1) As a **fraction**: $\frac{6 \cancel{\text{ft}}}{8 \cancel{\text{ft}}} = \frac{6}{8} = \frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 4} = \frac{3}{4}$
- 2) With a **colon**: $6 \text{ ft} : 8 \text{ ft} = 6 : 8 = 3 : 4$
- 3) With the word **to**: $6 \text{ ft to } 8 \text{ ft.} = 6 \text{ to } 8 = 3 \text{ to } 4.$

In a ratio, the **order** of the numbers is important! To write a ratio as a fraction,

- Place the **first** number in the **numerator**, place the **second** number in the **denominator**.
- Write the ratio in lowest terms – cancel common factors.
- A ratio compares two quantities with the **same** units. Convert the units if they are not the same.

Example 1: Write each ratio as a fraction in lowest terms.

- a) $1\frac{1}{2}$ miles (mi) to $2\frac{1}{4}$ miles (mi)

To write the ratio as a fraction in lowest terms, divide the numerator by the denominator and cancel common factors:

$$1\frac{1}{2} \text{ mi to } 2\frac{1}{4} \text{ mi} = \frac{1\frac{1}{2}}{2\frac{1}{4}} = 1\frac{1}{2} \div 2\frac{1}{4} = \frac{3}{2} \div \frac{9}{4} = \frac{\cancel{3}^1}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{\cancel{9}_3} = \frac{2}{3}$$

- b) 75 cents to \$1.25

The units are *not* the same. Since \$1 = 100 cents, convert \$1.25 to cents by moving the decimal point two places to the right: $\$1.25 = 1.25 = 125$ cents. This gives us:

$$75 \text{ cents to } \$1.25 = 75 \text{ cents to } 125 \text{ cents} = \frac{75}{125} = \frac{3 \cdot \cancel{25}}{5 \cdot \cancel{25}} = \frac{3}{5}$$

RATES

A **rate** is a comparison of two quantities with **different** units, such as the rate "10 grams per 180 milliliters." In a rate, the word "per" means "for each" and implies division. A rate is written as a fraction in lowest terms and the units are written as part of the rate. To write the rate 10 grams (g) per 180 milliliters (mL) as a fraction, cancel the corresponding 0's and keep the units:

$$10 \text{ g per } 180 \text{ mL} = \frac{\cancel{10} \text{ g}}{\cancel{180} \text{ mL}} = \frac{1 \text{ g}}{18 \text{ mL}}$$

A **unit rate** or **unit price** is a rate where the denominator is "1." For example, if 1 pound of coffee costs \$6.75, the unit price is written as:

$$\frac{\$6.75}{1\text{lb}} = \$6.75/\text{lb} \text{ or } \$6.75 \text{ per lb}$$

To find a unit rate or unit price, divide the numerator by the denominator.

Example 2: A car travels 330 miles on 15 gallons (gal). Find the unit rate.

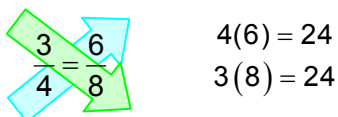
To find the unit rate, we divide the number of miles by the number of gallons:

$$330 \text{ miles on } 15 \text{ gallons} = \frac{330 \text{ mi}}{15 \text{ gal}} = 15 \overline{)330} = \frac{22 \text{ mi}}{1 \text{ gal}}$$

The unit rate is 22 mi/gal or 22 mpg.

PROPORTIONS

A **proportion** is a mathematical statement that two ratios or rates are equal. In a true proportion, the **cross products** are **equal**:



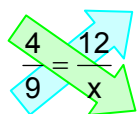
$$\begin{array}{l} 4(6) = 24 \\ 3(8) = 24 \end{array}$$

Because the cross products are equal, we can use them to solve a proportion when one of the numbers is unknown. To solve a proportion,

- 1) Cross multiply the ratios
- 2) Write an equation by setting the cross products equal to each other
- 3) Solve the equation for the variable.

Example 3: Solve $\frac{4}{9} = \frac{12}{x}$

To solve the equation for x, find the cross products and solve the resulting equation:



$$\begin{array}{l} 4 \cdot x = 9 \cdot 12 \\ 4x = 108 \\ \frac{4x}{4} = \frac{108}{4} \\ x = 27 \end{array}$$

← Cross multiply the ratios.

← Write an equation by setting the cross products equal to each other.

← Solve the equation for x.

SOLVING PROPORTION PROBLEMS

Proportions are used to solve a variety of problems, such as estimating wildlife populations, scaling distances on a map, or calculating mixtures and dosages.

Example 4: Use a proportion to solve the following problem:

"The brewing directions on a bag of ground coffee recommend 2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water. If the average-size coffee cup holds 6 oz of coffee, how many tablespoons of coffee are needed for eight 6-oz cups?"

To write the proportion,

- 1) **Identify the given ratio or rate and write it as a fraction.** In the problem above, we are given the rate "2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water." We can write this rate as:

$$2 \text{ tbs to } 6 \text{ oz} = \frac{2 \text{ tbs}}{6 \text{ oz}} = \frac{1 \text{ tbs}}{3 \text{ oz}}$$

- 2) **Assign a variable to represent the unknown quantity then write the second ratio or rate as a fraction.** Let x = the number of tablespoons we need to make eight 6-oz cups. Since each cup holds 6 oz, to make eight 6-oz cups, we need 48 oz of water. We can write this second rate as:

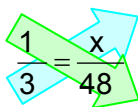
$$x \text{ tbs to } 48 \text{ oz} = \frac{x \text{ tbs}}{48 \text{ oz}}$$

- 3) **Write the proportion by setting the first ratio or rate equal to the second.** The order in which we write a proportion is important! *Always place like units across from each other.* Because the cross products are equal, inverting the ratios gives an *equivalent proportion*.

$$\frac{1 \text{ tbs}}{3 \text{ oz}} = \frac{x \text{ tbs}}{48 \text{ oz}} \quad \leftarrow \text{tbs across from tbs} \quad \text{or} \quad \frac{3 \text{ oz}}{1 \text{ tbs}} = \frac{48 \text{ oz}}{x \text{ tbs}} \quad \leftarrow \text{oz across from oz}$$

$\leftarrow \text{oz across from oz} \quad \leftarrow \text{tbs across from tbs}$

To find the number of tablespoons, cross multiply and solve the resulting equation:


$$3x = 48$$
$$\frac{3x}{3} = \frac{48}{3}$$
$$x = 16 \text{ tbs}$$

Hint: Alternatively, you may find it helpful to state the problem in the following way:

$$1 \text{ tbs is to } 3 \text{ oz} \quad \text{as} \quad x \text{ tbs is to } 48 \text{ oz}$$