Factoring Trinomials of the Form $x^2 + bx + c$

Trinomials, such as $x^2 + 5x + 6$ and $x^2 - 4x - 21$, are called trinomials of the form $x^2 + bx + c$ because the coefficient "squared" term is 1. In general, $\mathbf{x^2}$ is called the first or *leading* term, **bx** is called the *middle* term, and **c** is called the last or *constant* term. The steps we use to factor trinomials of this form are based on the patterns that occur when we FOIL binomials.

1) **<u>Arrange the terms in descending order</u>** (from highest power to lowest power) so that the expression takes the form: $x^2 + bx + c$.

2) **<u>Look for a GCF</u>** (Greatest Common Factor) and factor it out first. When the leading term is negative, the GCF is "−1." To factor out a gcf of −1, change the sign on each term.

3) **<u>Set up the first terms</u>**. Because $x^2 = x \cdot x$, the first term of each binomial factor is "**x**:"

$$(x \quad)(x \quad)$$

4) **<u>Determine the signs of the binomial factors</u>**.

   When the constant term is *positive*,

   - Binomial factors have the *same* sign: (x +)(x +) or (x −)(x −).
   - The middle term has the *same sign* as the binomial factors.
   - Factors of the constant term *add* to "**b**," the coefficient of the middle term.

   When the constant term is *negative*,

   - Binomial factors have *opposite* signs: (x +)(x −) or (x −)(x +).
   - Factors of the constant term **c** *subtract* to "**b**."
   - The *larger factor* of these factors has the *same sign* as the middle term.

5) **Find factors of "c" that sum to "b"** – list the factors of the constant term to find the pair of factors whose product is "**c**" and whose sum is "**b**." (**Note**: if factors of the constant term do not sum to "**b**," the expression is *"prime"* and cannot be factored.)

7) **Write the factorization** using the factors found in step 5.

6) **<u>Check the result</u>** by multiplying.

Example: Factor $2x + 8 - x^2$

Step 1: The terms of the expression are not in descending order. Rearrange the terms so that they take the form $x^2 + bx + c$.

$$2x + 8 - x^2 = -x^2 + 2x + 8$$

Step 2: Look for a **GCF** – because the leading term is negative, the GCF is −1. Factoring out the GCF gives us:

$$-x^2 + 2x + 8 = -\left(x^2 - 2x - 8\right)$$

Step 3: Next, factor the inside expression $x^2 - 2x - 8$. Because $x^2 = x \cdot x$, the first term of each binomial factor is "x:"

$$\begin{aligned} -x^2 + 2x + 8 &= -\left(x^2 - 2x - 8\right) \\ &= -(x \quad)(x \quad) \end{aligned}$$

Step 4: Determine the signs of the binomial factors. The constant term is *negative*. This means the binomial factors have *opposite* signs:

$$\begin{aligned} -x^2 + 2x + 8 &= -\left(x^2 - 2x - 8\right) \\ &= -(x + \quad)(x - \quad) \end{aligned}$$

Step 5: Find the factors of –8 that sum to –2, the coefficient of the middle term. Because the middle term is negative, the larger factor of –8 must be negative. Factors of –8 that sum to –2 are **2** and **–4**.

Factors of –8	Sum of Factors
1 • –8;	1 – 8 = –7
2 • –4	2 – 4 = **–2**

Step 6: Write the factorization using the factors found in Step 5. Don't forget to write the GCF as part of the factorization!

$$\begin{aligned} -x^2 + 2x + 8 &= -\left(x^2 - 2x - 8\right) \\ &= -(x + 2)(x - 4) \end{aligned}$$

Step 7: To check the result, multiply the binomial factors first then distribute the GCF:

$$-\left(x + 2\right)\left(x - 4\right) = -\left(x^2 - 4x + 2x - 8\right) = -\left(x^2 - 2x - 8\right) = -x^2 + 2x + 8$$